Slender Wings with Leading-Edge Vortex Separation: A Challenge for Panel Methods and Euler Solvers

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The demands of performance and maneuverability on modern fighter aircraft and missiles require slender wings, whose leading edge vortex flows result in increased lift at high angles of attack This paper reviews the history of the utilization of these vortex flows, and the development of theoretical methods for prediction of the nonlinear characteristics of slender wings Possible improvements based on the integral physical effects of the vortex flow are also considered Recent advances in the numerical solution of the time dependent Euler equations make Euler methods an attractive alternative to the methods based on potential flow solution ap proaches The latter are usually limited to subsonic speeds and demand as much effort to predict the flow around more complex configurations as do the Euler methods The latter even are able to predict flow separation effects to some extent, providing significant support for the evaluation of more complex shapes, thus improving the design procedures Results obtained by a panel procedure and by an Euler method are presented for different wings and wing body configurations Preliminary results obtained by applying these methods to vortex breakdown are included

I Introduction

FOR a slender wing designed for high speed performance, the leading edge is quite sharp, causing flow separation along the leading edge even at low incidence The separated vortex sheets roll up to form spiral shaped primary vortices above the wing's upper surface (Fig 1) Due to the induced velocities, the pressure distribution (Fig 2) has a distinct minimum beneath the vortex axis The associated lift increase causes the well known nonlinear characteristics exhibited by slender wings (Fig 3)

The steep pressure gradient between the pressure minimum and the leading edge causes boundary layer separation and an associated small secondary vortex As the considered methods do not regard viscous effects, and as these secondary vortices have only limited influence[1] on the overall flowfield, this effect will not be described or discussed here At very high angle of incidence a rapid change of the vortex structure occurs above the wing, which results in a decrease in lift and a nose-up pitching moment, limiting maneuverability This "vortex bursting" moves forward from the trailing edge to the apex with increasing incidence, causing loss of lift on the upper surface.[2]

II Performance and Maneuverability of Slender Wings with Leading-Edge Vortices and Development of Theoretical Methods

Although the vortex formation on slender wings was already investigated in some early experiments by Winter[3] in 1935, its application was not considered until slender wings became a new design tool

When the possibility of transonic and supersonic flight was seriously considered, the results of aerodynamic research[4] showed that swept wings, especially pointed delta wings of small aspect ratio, could have lower wave drag at supersonic speed than did unswept wings A theoretical approach revealed that a subsonic leading edge was a possible means for supersonic drag reduction Also very important for a supersonic configuration was the use of thin wing sections, and the delta planform turned out to be the best structural shape Consequently, most of the first supersonic aircraft were deltas (Fig 4) Low-aspect ratio wings having a poor lift derivative were expected to present difficult problems in the landing approach and during maneuvering flight. Studies in 1946 of the DM-1 glider research plane[5,6] (Fig 4) revealed, more or less accidentally, the effect of leading edge (LE) vortices The restricted distribution of these results allowed the rediscovery of the LE vortices in 1951, when unexpected vortices, peeling off the wings of a version of an F 86 fighter (Fig 4), provided extra lift This led to some modifications in which for the first time leading edge vortices were used to roughly double the combat maneuverability.[7] At the same time, in Europe tests of some very slender wings in an ONERA wind tunnel revealed a strong vortex flow on the surface of an arrow wing [8] The researchers at Royal Aircraft Establishment[9] discussed the constructive role of controlled leading edge separation to improve low speed characteristics of slender wings

First knowledge was obtained from experiments The leading edge vortex flow assisted, for instance, the F 102 and Mirage-type aircraft (Fig. 4) at low speeds A first conical model of the flow was proposed by Legendre[10] by superimposing a pair of vortices above the wing (Fig 5a) and satisfying a Kutta condition at the leading-edge However, as Adams[11] discussed, the solution concerning the lift and pressure distribution of Legendre's model was not unique Edwards[12] proved that for conical slender wings a reasonable mathematical model must also consider the mechanism of the feeding vortex-sheet emanating from the wings leading edge The location of the complete vortex system is determined from the condition that it must remain force-free Brown and Michael[13] applied the feeding-sheet vortex system in the crossflow plane (Fig 5a) Some ten years later, the supersonic transport programs (Fig 4) offered exceptional opportunities to look at the vortex flow in more detail Intensive flight tests[14,15] and a great deal of experimental work,[1,2,16] provided a more detailed description of leading edge vortices, their structure, and their consequences (Figs, 1 3) The analyses pointed out a very severe limitation of vortex stability, the

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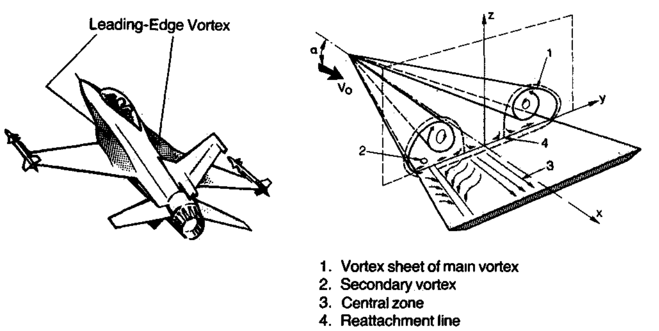

Fig. 1 Leading-edge vortex and its structure.

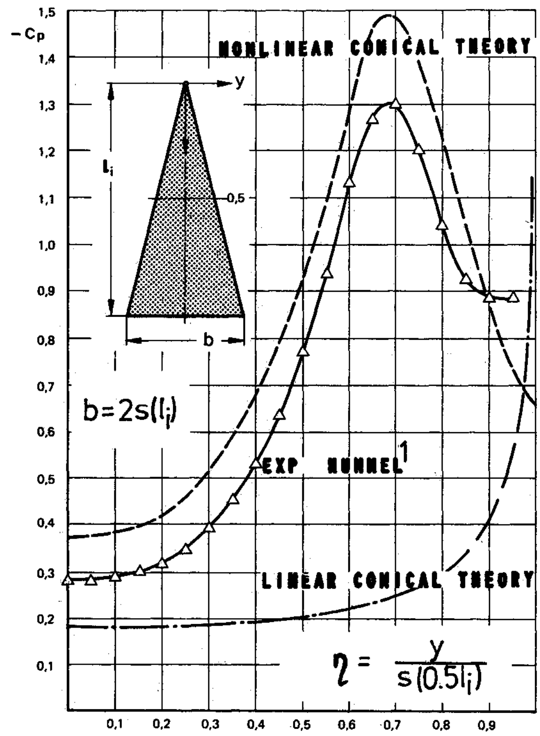

Fig. 2 Pressure distribution.

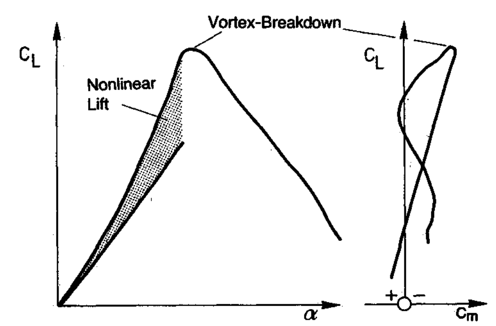

Fig. 3 Characteristics of the aerodynamic derivatives.

previously mentioned, so-called vortex breakdown. In recent years a number of different breakdown explanations have been proposed, a partial list of which is considered in Ref. 17 This phenomenon is discussed further in Sec. IV

During this period, Mangler and Smith[18] published the best known model to simulate the vortex system. By the introduction of a free vortex sheet (Fig. 5a) between the wing's leading edge and the feeding sheet, they overcame the deficiencies of the Brown and Michael method, which was unable to satisfy the full Kutta condition along the leading edge. By demanding the free sheet to be a stream surface and assuming conical flow, they were able to predict the pressure distribution that had been observed experimentally on the front part of the delta wing (Fig. 2). Towards the trailing edge the model overpredicts the load distribution because the Kutta condition cannot be satisfied at the trailing edge by the conical theory.

A very suitable method for calculating the magnitude of the nonlinear vortex lift of a rather broad class of slender wing planforms is provided by the leading-edge suction analogy. Polhamus[19] reasoned that the normal force needed for the separated flow around a leading edge to attach to the wing is equivalent to the suction force necessary to force the flow to be attached to the leading edge in an unseparated condition (Fig. 5b). Unfortunately, this method gives no flowfield details or surface pressure distributions.

While combat aircraft design during that time was centered strictly around demands of performance, the analyses of combat experience resulted in the development of aircraft which were more able to meet the maneuverability requirements of dogfights because of lower wing loading and improved engines. Corresponding designs such as the F-16 (Fig. 4), reconsidered the explotation of vortex lift to enhance maneuverability. Nevertheless, current ideas on future mission demands require a proper balance between both maneuverability and performance. This points towards carefully designed delta wing configurations and their derivatives (Fig. 4), which exploit the lift-improving effects of the vortex flow while avoiding the penalties associated with its effects on lateral stability and drag.

While design application of numerical methods in the early 1970's was restricted to simple lifting-line theories and panel methods, currently new tools are available to improve the aircraft design procedures. Higher-order panel methods[20] make it possible to translate the physics of Mangler and Smith's model into a fully three-dimensional, extremely exact method (Fig. 5b).[21,22] Along with the suction analogy and some other vortex lattice-based methods, the three-dimensional problem is now solved in principle (Fig. 6). However, the higher-order panel methods are not yet fully satisfactory due to the high computational expense required for only limited capability, as they are restricted to subsonic speeds. For simple shapes, this points towards the development of a simplified panel model which relies on the integrated flow effects to provide a computational method as capable as the higher-order procedure. Separation on double-deltas or more complex strake wing configurations require very elaborate panel or vortex-lattice methods.[23-25] The simulation of multivortex flows by potential flow methods demand careful numerical treatment concerning separation lines and the assumed vortex interaction. Current capabilities of existing codes need further examination.

An alternative to the potential flow method is to use the time-dependent Euler equations. These have more geometric versatility, and the more difficult configuration can be considered with little difficulty. Due to the nature of the Euler equations, Euler methods are not limited to subsonic Mach numbers but are valid through the transonic regime up to supersonic speeds. How they will help to compute leading-edge vortex flow will be described in the next section.

III. Simplified Models and Euler Method to Describe Leading-Edge Vortex Flows

Potential Flow Methods

The idea of a simplified method is based on the approaches of Mangler and Smith[18] and Brown and Michael.[13] In Ref. 26 a planar free vortex sheet is introduced in addition to the bound vortex sheet above the wing (Fig. 7a) which is different from the rolled up free vortex sheet in Ref. 18 (Fig. 7b). As in Mangler and Smith's model the flowfield is assumed to be conical. However, no flow is permitted through the free vortex sheet. Instead of applying the usual boundary con-

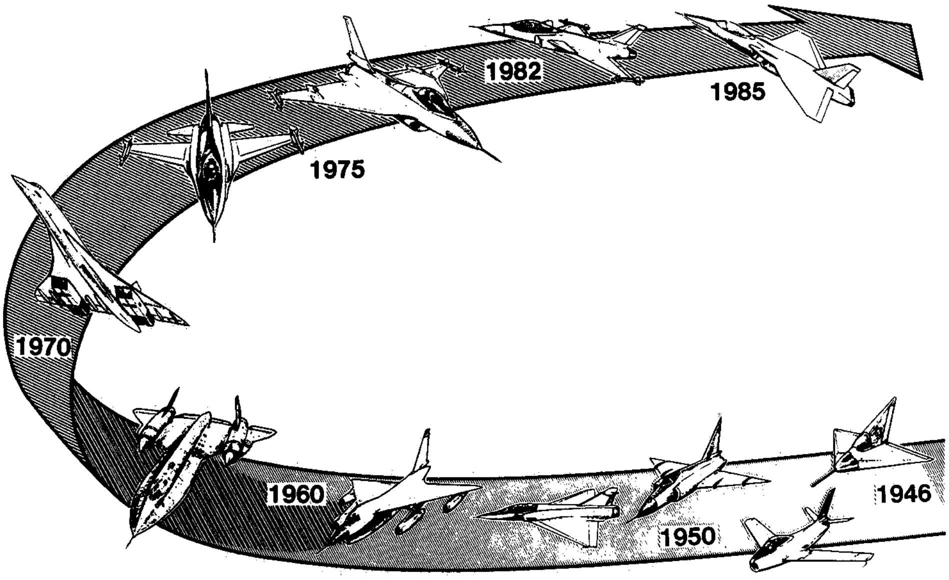

Fig 4 History of application

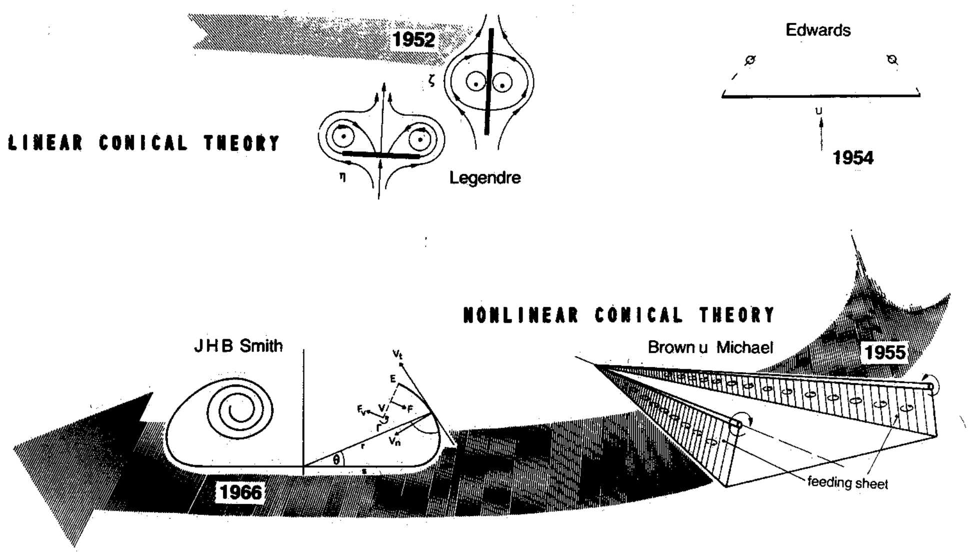

Fig 5a Historical development of leading edge vortex models, early development

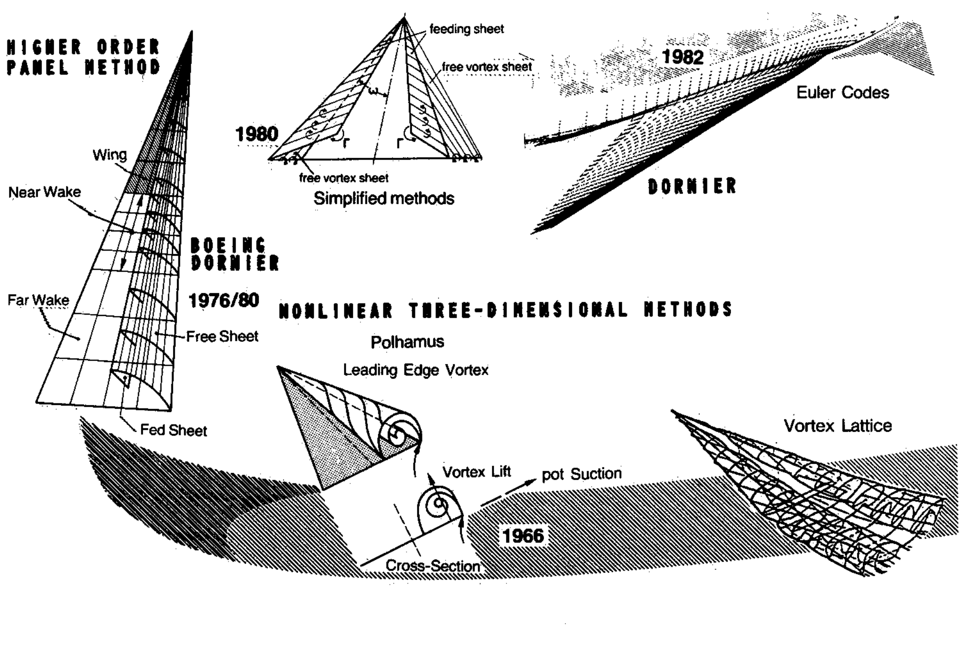

Fig 5b Historical development of leading-edge vortex models, recent development

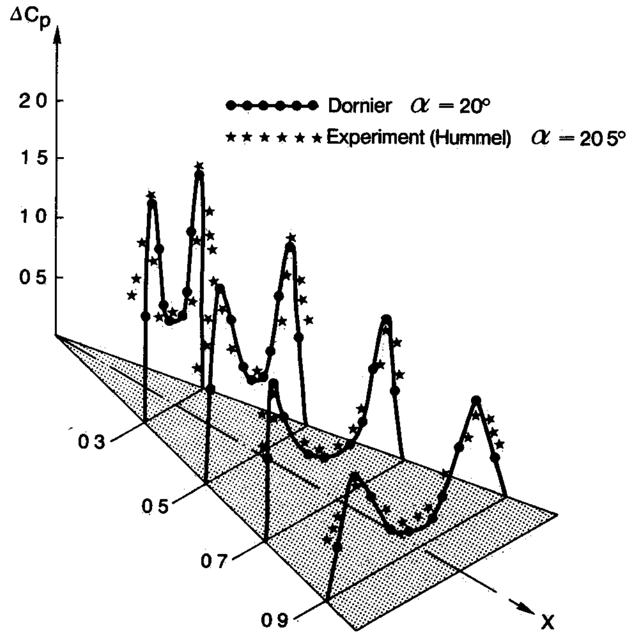

Fig 6 Results of a higher order panel method.

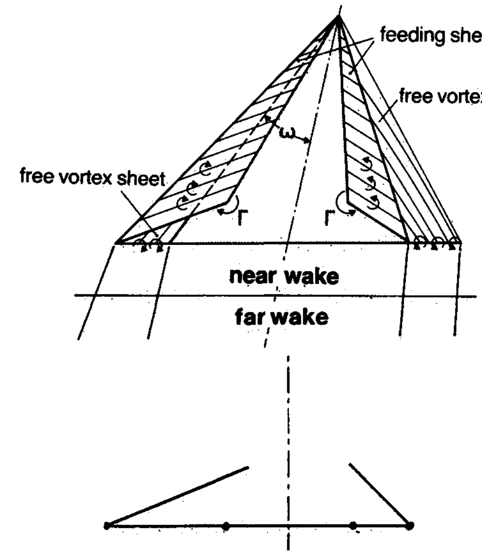

Fig 7a Three dimensional panel methods, two alternative simplified panel methods

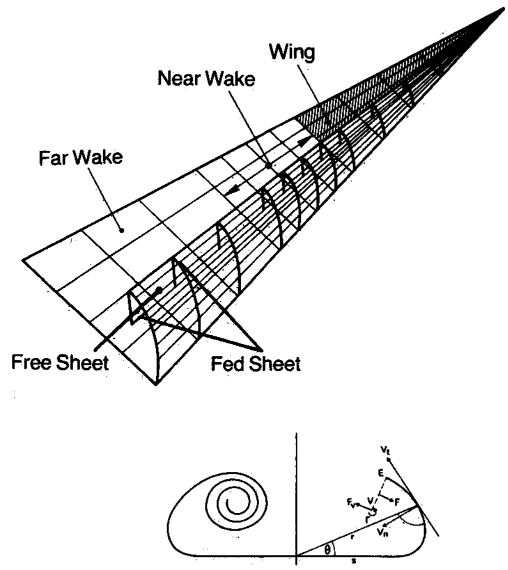

Fig 7b Three dimensional panel methods, higher order panel method.

dition to free vortex sheets, the suction analogy[19] is fulfilled by prescribing normal force coefficients and the condition that the whole feeding system should be force-free One parameter is free and can be used for optimum adaptation of the results to those of Mangler and Smith's method, or for making the vortex sheet free of forces immediately at the leading edge This procedure provides satisfactory results for the pressure distribution (Fig 8). Further investigations evidently revealed the simplification of Mangler and Smith's model to Brown and Michael's by shortening the free vortex sheet. Moreover, the results demonstrated the importance of this sheet, which can be explained by its near-field influence close to the leading edge, permitting a more appropriate fulfillment of the Kutta condition

The three-dimensional effect is determined primarily by the model of the trailing-edge Kutta condition in all three dimensional procedures based on conical cross-flow models. By coincidence of vortex and velocity vectors, each free vortex sheet becomes a real stream surface, the same as it is for a wake model Employing the vector algebra on the formula for the aforementioned condition will disclose two subconditions

$$V \times \gamma = n\ (V \operatorname{grad} \mu) - \operatorname{grad} \mu\ (Vn) = 0$$

since $n \times \operatorname{grad} \mu$ describes the vorticity γ

Proper solution of the equation demands that both parts become zero independently because they are not unidirectional The first term corresponds to the condition of no-load, the second one to the kinematic flow condition. Immediately behind the wing the assumption of a geometrically "frozen" wake[22 27] seems to be justified, and by meeting the first subcondition $\Delta Cp = 0$ normal to the wake, the Kutta condition can be fulfilled appropriately Due to the fixed geometry the wing's influence coefficients remain unchanged, and only the coefficients of the nonlinear ΔCp equations cause iterative computational effort Figure 7b shows the location of the so called Kutta panels along the wing's trailing edge Here the induced velocities, taken from the previous iteration according to Biot Savart's law, create forces which must vanish perpendicular to the panel's plane This condition is enforced by the corresponding equations and the adaption of the solution of the next iteration The complete simplified procedure, the connection of the Kutta and the leading edge models, is now under development The preliminary results look promising, and the computational means will be available in the near future

Euler Methods

In recent years great advances have been achieved with the numerical solution of the three dimensional Euler equations According to Ref 28, the time-dependent Euler methods can predict separation phenomena, despite neglecting viscosity, if supersonic speed occurs locally somewhere at the considered configuration during the transient computation The shock induced losses of total pressure and the subsequent vorticity production allow the formation of vortex sheets emanating from the leading edge during the time-marching procedure The converged solution does not necessarily show any remaining supersonic regimes This time-marching method needs no specified Kutta condition, as it true for any space-marching Euler methods, for example the Euler method described in Refs 29 and 30 The physical laws describing conservation of mass, momentum, and energy are written in integral form for unsteady, compressible flow

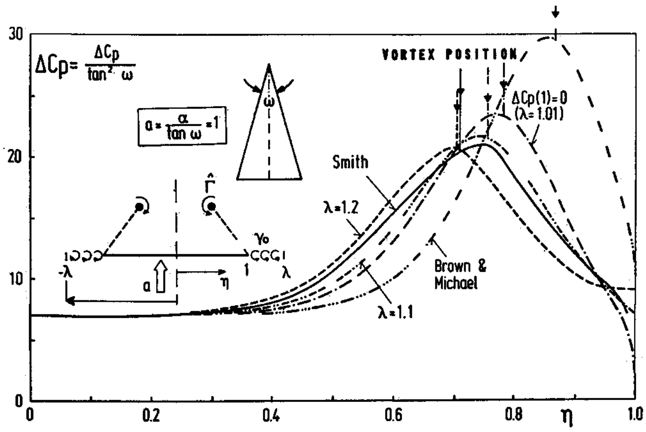

Fig. 8 Simplified conical theory (pressure distribution).

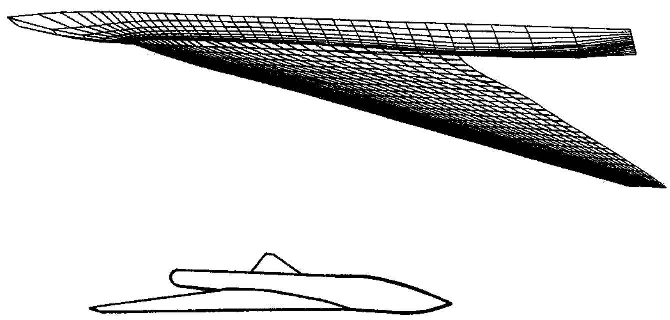

Fig. 9 Euler method network.

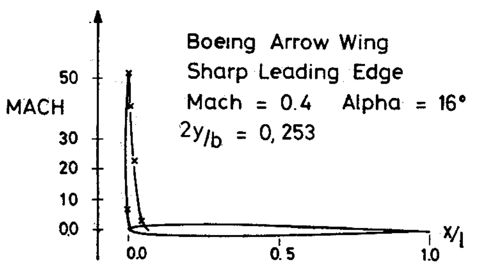

Fig. 10 Mach number distribution computed by the solution of the full potential equations.

$$\int_V \frac{\partial \rho}{\partial t} \mathrm{d}V + \int_S \rho \mathbf{vn} \mathrm{d}S = 0 \tag{1}$$

$$\int_V \frac{\partial (\rho \mathbf{v})}{\partial} \mathrm{d}V + \int_S \mathbf{v}(\rho \mathbf{vn}) \mathrm{d}S + \int_s p\mathbf{n} \mathrm{d}s - \int_S T\mathbf{n} \mathrm{d}s = 0$$

$$\int_V \frac{\partial E}{\partial t} \mathrm{d}V + \int_S (E+p)\mathbf{vn} \mathrm{d}S - \int_S \mathbf{v}(T\mathbf{n}) \mathrm{d}S + \int_S \mathbf{Qn} \mathrm{d}s = 0.$$

In addition to the boundary conditions at the edge of the computational domain, only the condition of no flow through solid surfaces must be satisfied. Details of the method can be found in Refs. 31 and 32. As the accuracy of these finite volume approaches depends mainly on the representation of the geometric configuration, contour-conformal meshes are used for the discretization (Fig. 9). Reference 33 gives a survey of strategies for these mesh-generating techniques. When the computed flow is being forced around sharp corners and edges, as in the case of the wing-body configuration depicted in Fig. 9, very high supersonic velocities are encountered near wing leading and trailing edges, even at subsonic freestream velocities. Figure 10 shows the result of a potential flow solution, which may be considered as a first computational cycle of a Euler method. The figure reveals that extremely high Mach numbers are computed close to the leading edge. As the Euler method includes compressible flow effects it responds with a shock to provide a fit to the surrounding flow Three effects must be considered: Crocco's Theorem, numerical total pressure losses due to approximations in the boundary conditions at highly curved surfaces, and the numerical viscosity of the filter step of the employed scheme. According to Crocco, rotation is forced by the rules of change of entropy across a shock of variable strength. Recalling Kelvin's law, it appears conceivable that this induced vorticity is conserved even when convergence is obtained without any supersonic regions remaining. Also, the filter step may contribute to the production of rotational flow However, careful two- and three-dimensional studies on simplified realistic test cases[28,31] performed for an AGARD working group prove the effects of the numerical viscosity to be minor. On the other hand, numerical total pressure errors due to the numerical formulation for solid surface boundary conditions can be significant if the mesh spacing at highly curved surfaces is not appropriate. While this will not change any result for sharp edges, the round leading edge results can depend strongly on mesh spacing determining whether or not separated flow is obtained for a given angle of attack. Figure 11 gives examples of the pressure distribution on the configuration depicted in Fig. 9 for two different leading edge forms (sharp and round) and a subsonic and a transonic Mach number. The part span separation is reproduced in Fig. 12 from the isobars of the arrow wing with the round leading edge exposed to an 8-deg angle of attack. The comparison of the computed results is very well in agreement with experimental results. At the lower incidence the typical isobar pattern for separation is visible only at the rear part of the wing, while at 16 deg the same pattern is found everywhere along the leading edge, and is even more pronounced at $\alpha = 35$ deg. Figure 13 depicts the good agreement between experimental and theoretical lift coefficients.

Since they are not restricted to any speed regime such as panel methods or vortex lattice procedures, the Euler methods are applicable from subsonic through transonic to supersonic speeds without any modifications. Apart from this, no fundamental limitation is posed on future configurations (Fig. 4). In contrast, panel methods, etc., normally are

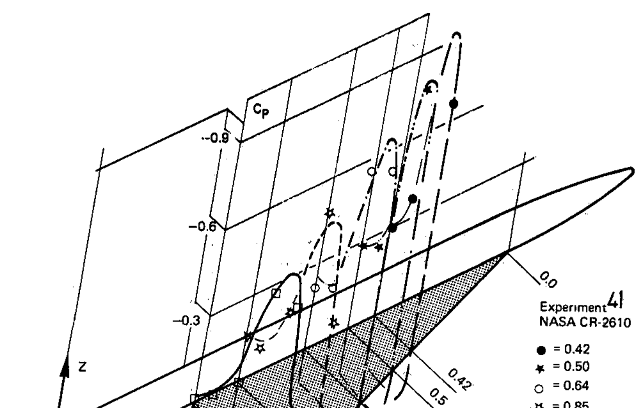

Fig. 11a Pressure distribution computed by a Euler method for a body wing configuration at $\alpha = 16$ deg; sharp leading edge, M = 0.4.

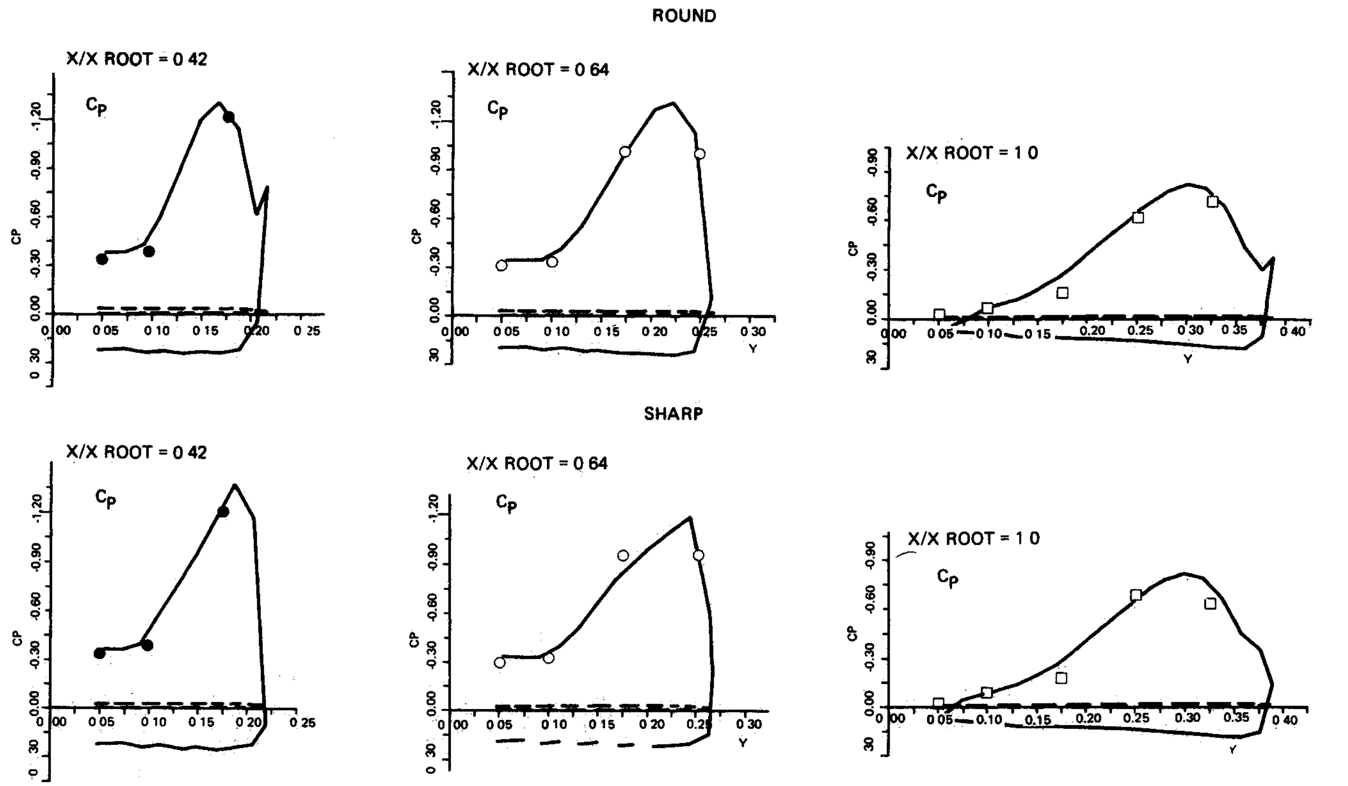

Fig 11b Pressure distribution computed by a Euler method for a body wing configuration at $\alpha = 16$ deg; effect of leading edge roundness, M = 0 4

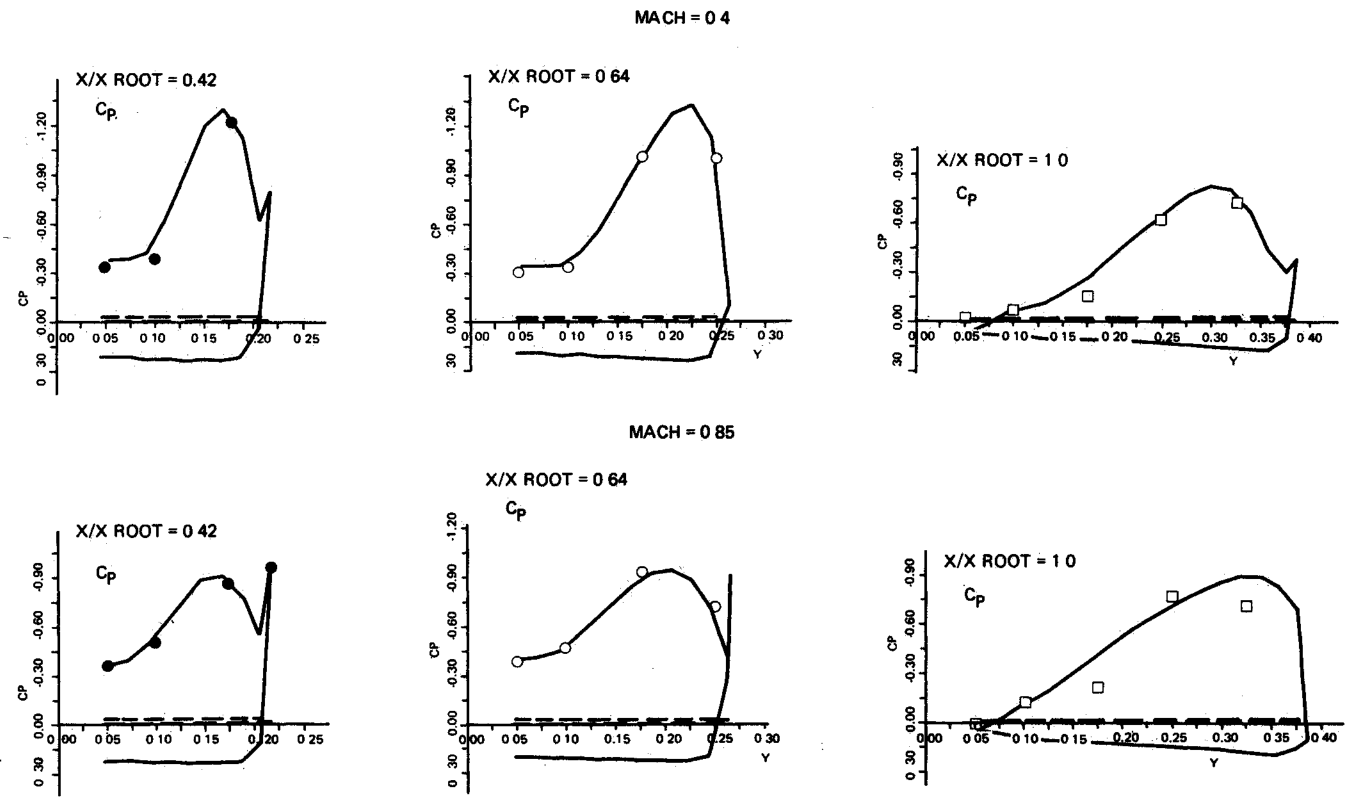

Fig 11c Pressure distribution computed by a Euler method for a body wing configuration at $\alpha = 16$ deg; effect of subsonic Mach number, rounded leading edge

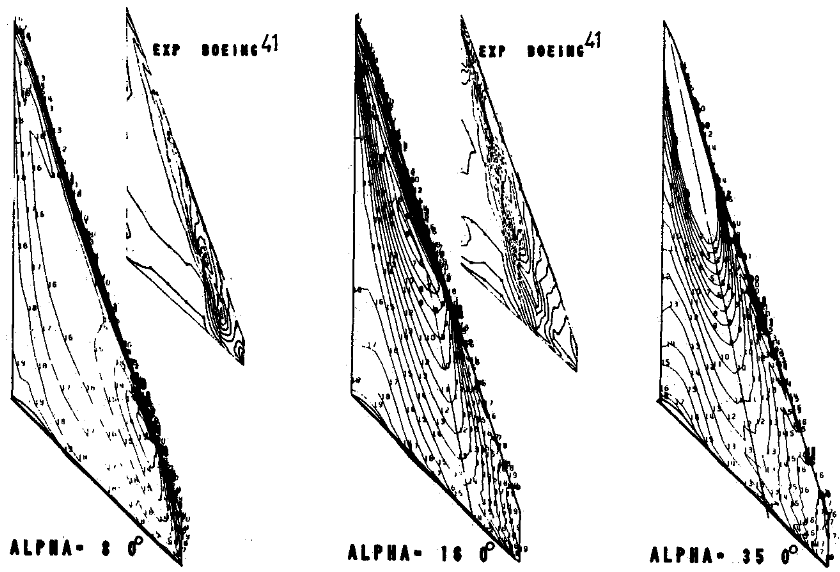

Fig. 12 Isobars on the leeside wing surface.

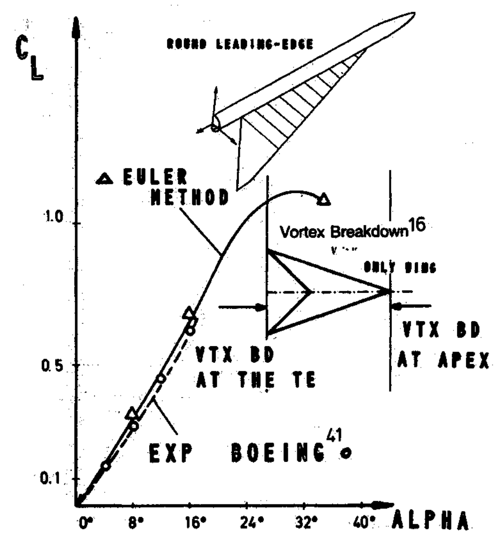

Fig. 13 Lift derivatives of the body wing configuration.

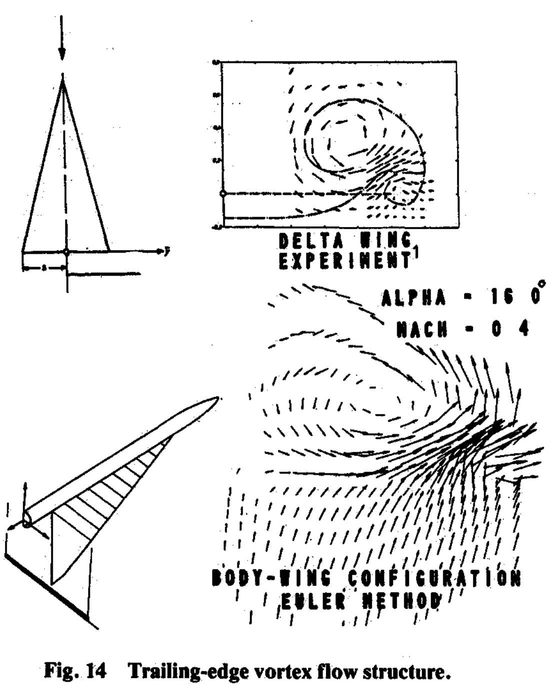

Fig. 14 Trailing-edge vortex flow structure.

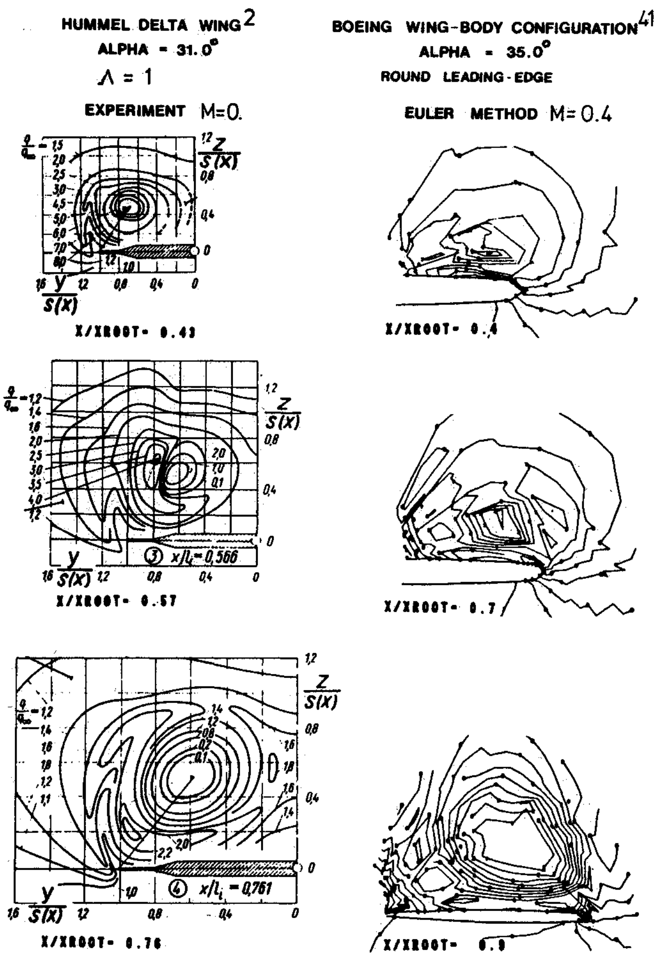

Fig. 15 Dynamic pressure above a wing at breakdown conditions.

restricted to one pair of vortices and demand significant modifications and manipulations for analysis of complex geometries, i.e., when the structure of the vortex systems cannot be described in easy terms.[23] Thus, in the future, Euler methods will be used for the investigation of more complex configurations. However, the panel and vortex lattice methods will not cede their importance as pilot codes for preliminary design methods and for further investigations concerning the vortex structure, such as in the vortex-breakdown phenomenon.

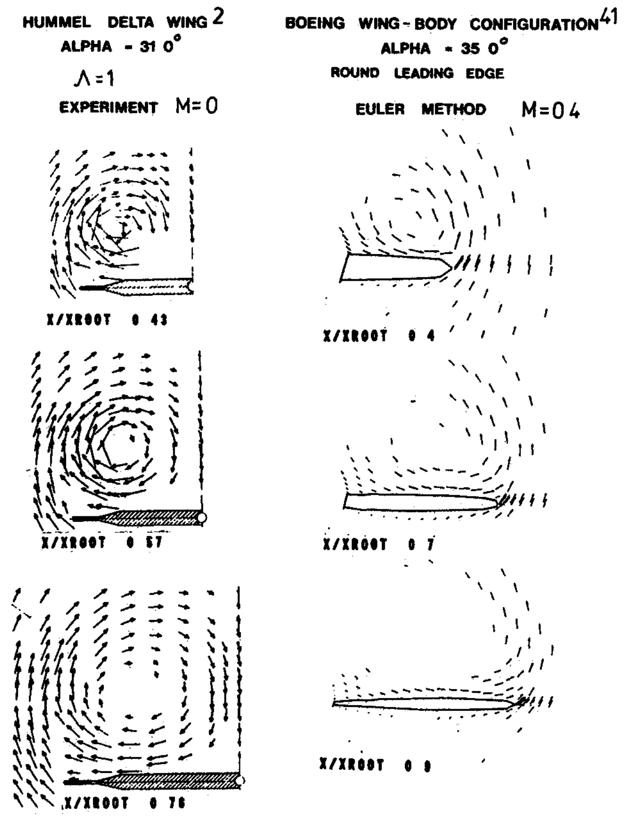

Fig 16 Velocity vector field above a wing at breakdown conditions

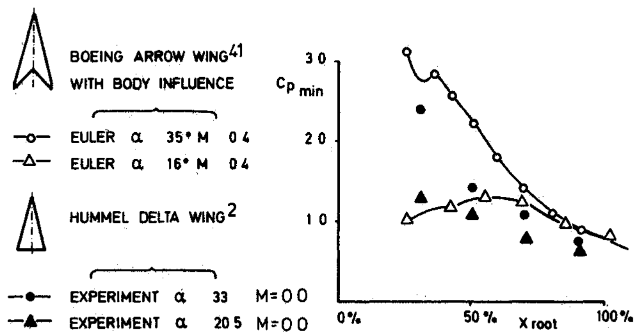

Fig 17 A trend comparison of the minimum pressure underneath the vortex path at vortex breakdown conditions

IV Some Ideas on Vortex Breakdown

At very high angles of attack a rapid change of the vortex structure, a so-called vortex breakdown, occurs above the wing The results of Hummel's experiments[2] unveiled the important influence of the pressure gradient along the vortex path An increasing pressure promotes breakdown, a decreasing pressure stabilizes the vortex structure Upstream of the breakdown point the loss of total pressure is limited to a small area around the vortex axis, while downstream losses are spread across the whole core region (Fig 14) Facing a positive pressure gradient, the axial core velocity in the vortex, which may be very much higher than the velocity of the surrounding flow, decelerates, sometimes until flow reversal occurs Now the mass flux along the axis can be sustained no longer without a widening of the core to fit the flow to the downstream conditions

In recent years a number of different explanations have been proposed The following partial list is considered in Ref 17 Ludwieg[34] derived his work from Rayleigh's[36] instability examinations of the flow between two coaxial rotating cylinders Benjamin[37] considers the analogy with the hydraulic jump or the shockwave in describing flow conditions behind the breakdown position, while Hall[38] tries to explain the problem by developing a boundary layer analogy within the vortex core

It is evident that all the theories mentioned describe important parts of the breakdown procedure, and it is possible that using a combination of them in further theoretical examinations will get closer to the facts A review[17 39] of the different experimental and theoretical work on breakdown leads to the conclusion that this phenomenon is in principle not governed by viscous effects but by effects describable by inviscid procedures such as potential flow and Euler methods [40] In Ref 41 the very first tests with a higher order panel procedure are described Unfortunately, because of deficiencies in this method the final results must be interpreted carefully The panel method of vortex lattice type described here, however, provides a model based on the integral physical effects and should be well suited for stability investigations of the vortex system With the employment of different vortex core models, the results will help to outline a simple criterion or procedure to determine the conditions for beginning vortex breakdown above the wing

Figures 15-17 depict the results of present trials to determine the breakdown phenomenon using a Euler procedure Unfortunately, no compatible measurements have been performed until now However, the comparison of the computational results for the arrow wing configuration[42] at high angle of attack and Hummel's delta wing investigations (Ref 2) show similar tendencies As in the experimental results, the computed dynamic pressure losses behind breakdown spread across the whole core region (Fig 15) The velocity vector plots (Fig 16) describe the same effect The behavior of the lift coefficient also clearly reveals the reduced lift due to breakdown (Fig 13). Hummel's experiments (Ref 2) show a significantly decreasing suction peak behind the breakdown position to be responsible for this reduced lift This is also visible in Fig 17 Here the Euler results for the body-arrow wing configuration show similar tendencies to Hummel's vortex breakdown investigations (Ref 2) Further investigations are planned for the near future to improve our knowledge of the complex vortex breakdown phenomenon

V Conclusions

Due to computer and algorithm improvements, Euler methods may become the most powerful tool for detailed investigations of complex configurations Panel and vortex lattice procedures, however, will have a principal future use in detailed basic research of vortex flows For instance, the mechanism of vortex breakdown might be revealed by relatively simple stability investigations Euler methods, together with corresponding evaluation procedures, permit a survey of the complete flowfield for complex shapes at all typical flight regime Mach numbers Thus, complicated geometries, such those of military aircraft, can be analyzed throughout their complete flight envelope, from subsonic through transonic and supersonic speeds, using one computational tool Together, both approaches can pilot the way to a better understanding of the dynamics of controlled separated flows The Euler method especially will help limit the wind tunnel workload for validation of future aircraft designs

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